**Question 1: (2+3+5+5)**

An organization wants to conduct a mental test for its employees in which the questioner contains multiple dichotomous questions. The organization wants to estimate proportion (p) of an examinee's correct score. Suppose they heir you for the task. You select an employee at random and found that he/she answers 65 correct answers out of 90 questions chosen randomly from a given set of questions.

**a)** Given p, what is the probability distribution of the random variable: X= “number of correct answers by an employee in the sample of size n”? Based on classical statistics, compute a 99% confidence interval for p.

**Answer a) :** Here let us first analyze the nature of distribution for X,

**X=“number of correct answers by an employee in the sample of size n”**

where n is the number of questions answered.

Since each question is answered either correctly or incorrectly (dichotomous), X follows a binomial distribution.

The probability distribution of X, given p, is described by the binomial probability mass function:

**P(X=k) = (nCk )x (p^k) x ((1-p)^(n-k))**

* n is the number of questions answered
* k is the number of correct answers
* p is the probability of answering a question correctly

To compute a 99% confidence interval for p, The formula for the confidence interval is:

**Confidence interval =(p - z \*(p\*(1-p))/squareroot(n) , p + z \*(p\*(1-p))//squareroot(n))**

Lets Calculate,

p= 65/90 =0.7222

z for 99 % confidence interval is 2.567

there confidence interval on substituting values is

**CI=**((0.722-2.567\*(0.722\*(0.278))/(9.486)) , (0.722+2.567\*(0.722\*(0.278))/(9.486)))

**CI=** (0.652,0.791)

So, the 99% confidence interval for the proportion of **correct answers p is approximately (0.652, 0.791).**

**b)** Considering a uniform distribution prior for p. What will be the posterior distribution of p and 99% credible interval for p. Comment on the similarity (or not) with the interval obtained in part (a).

**Answer b) :**

To find the posterior distribution of p and the 99% credible interval for p, we can use Bayesian Approach.

Given a uniform distribution for prior **p, the posterior distribution will be proportional to the likelihood function.**

The **likelihood function follows a binomial distribution**, and the uniform prior distribution is **constant across the range [0, 1].** Therefore, the posterior distribution will be proportional to the likelihood function.

**f(p|data) ∝ f(data|p) ⋅ P(p)**

Where:

* data represents the observed data
* P(data∣p) is the likelihood function (binomial distribution)
* P(p) is the prior distribution (uniform distribution)

*f*(*p*∣data)∝*p*⋅(1−*p*)25

c) Based on a previous study by an expert, suppose the prior distribution for p is obtained to be beta (2,4). Based on the posterior distribution, what is the estimate, sd and 99% credible interval for p?

**Answer c):**

d) Suppose additional 8 questions were answered. Based on the result of part (c), compute the predictive probability distribution for the number of questions in this new sample which are answered correctly

**Answer d):**